

On Detection Issues in the SC-based Uplink of a MU-MIMO System with a Large Number of BS Antennas

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Abstract—This paper deals with Single Carrier (SC)/Frequency Domain Equalization (FDE) within a Multi-User (MU)-Multi-Input Multi-Output (MIMO) system where a large number of Base Station (BS) antennas is adopted. In this context, either linear or reduced-complexity iterative Decision-Feedback (DF) detection techniques are considered. Regarding performance evaluation by simulation, appropriate semi-analytical methods are proposed.

This paper includes a detailed evaluation of Bit Error Rate (BER) performances for uncoded 4-Quadrature Amplitude Modulation (4-QAM) schemes and a MU-MIMO channel with uncorrelated Rayleigh fading. The accuracy of performance results obtained through the semi-analytical simulation methods is assessed by means of parallel conventional Monte Carlo simulations, under the assumptions of perfect power control and perfect channel estimation. The performance results are discussed in detail, with the help of selected performance bounds. We emphasize that a moderately large number of BS antennas is enough to closely approximate the Single-Input Multi-Output (SIMO) Matched-Filter Bound (MFB) performance, especially when using the suggested low-complexity iterative DF technique, which does not require matrix inversion operations. We also emphasize the achievable "massive MIMO" effects, even for strongly reduced-complexity linear detection techniques, provided that the number of BS antennas is much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users.

I. INTRODUCTION

Cyclic-Prefix (CP)-assisted block transmission schemes were proposed and developed, in the last two decades, for broadband wireless systems, which have to deal with strongly frequency-selective fading channel conditions. These schemes take advantage of current low-cost, flexible, Fast Fourier Transform (FFT)-based signal processing technology, with both Orthogonal Frequency Division Multiplexing (OFDM) and SC/FDE alternative choices [1], [2], [3]. Mixed air interface solutions, with OFDM for the downlink and SC/FDE for the uplink, as proposed in [2], are now widely accepted; the main reason for replacing OFDM by SC/FDE, with regard to uplink transmission, is the lower envelope fluctuation of the transmitted signals when data symbols are directly defined in the time domain, leading to reduced power amplification problems at the mobile terminals.

The development of MIMO technologies has been crucial for the "success story" of broadband wireless communications in the last two decades. Through spatial multiplexing schemes, following and extending ideas early presented in [4], MIMO systems are currently able to provide very high bandwidth efficiencies and a reliable radiotransmission at data rates beyond 1 Gigabit/s. Appropriate MIMO detection schemes, offering a range of performance/complexity tradeoffs [5] - and also joint iterative detection and decoding schemes [6], have been essential for the technological improvements in this area. In the last decade, MU-MIMO systems have been successfully implemented and introduced in several broadband communication standards [7]; in such "space division multiple access" systems, the more antennas the BS is equipped with, the more users can simultaneously communicate in the same time-frequency resource.

Recently, the adoption of MU-MIMO systems with a very large number of antennas in the BS, much larger than the number of mobile terminal (MT) antennas in its cell, was proposed in [8]. This "massive MIMO" approach has been shown to be recommendable for several reasons [8], [9], [10]: simple linear processing for MIMO detection/precoding (uplink/downlink), namely when using OFDM for broadband block transmission, becomes nearly optimal; both Multi-User Interference (MUI)/Multi-Stream Interference (MSI) effects and fast fading effects of multipath propagation tend to disappear; both power efficiency and bandwidth efficiency become substantially increased.

This paper deals with SC/FDE for the uplink of a MU-MIMO system where the BS is constrained to adopt low-complexity detection techniques but can be equipped with a large number of receiver antennas. In this context, either a linear detection or a reduced-complexity iterative DF detection are considered. As to the linear detection alternative, we include both the optimum Minimum Mean-Squared Error (MMSE) [11] and the quite simple Matched Filter (MF) detection cases. The iterative DF detection alternative, which resorts to joint cancellation of estimated MUI/MSI and Inter-Symbol Interference (ISI), does not involve channel decoding, differently from the iterative receiver technique of [6]; it can be regarded as an exten-

sion to the multi-input context of the reduced-complexity iterative receiver techniques previously considered for SIMO systems by the authors (see [12], [13], [14] and the references therein).

Regarding performance evaluation by simulation, appropriate semi-analytical methods are proposed, combining simulated channel realizations and analytical computations of BER performance which are conditional on those channel realizations; selected analytical and semi-analytical performance bounds and a simple characterization of "massive MIMO" effects are also provided. This paper shows and discusses a set of numerical performance results. The main conclusions of the paper are presented in the final section.

II. SYSTEM MODEL

A. SC/FDE for MU-MIMO Uplink Block Transmission

We consider here a CP-assisted SC/FDE block transmission, within a MU-MIMO system with N_T TX antennas and N_R RX antennas; for example, but not necessarily, one antenna per Mobile Terminal (MT). We assume, in the j th TX antenna ($j = 1, 2, \dots, N_T$) a length- N block $\mathbf{s}^{(j)} = [s_0^{(j)}, s_1^{(j)}, \dots, s_{N-1}^{(j)}]^T$ of time-domain data symbols in accordance with the corresponding binary data block and the selected 4-QAM constellation under a Gray mapping rule. The insertion of a length- L_s CP, long enough to cope with the time-dispersive effects of multipath propagation, is also assumed.

By using the frequency-domain version of the time-domain data block $\mathbf{s}^{(j)}$, given by $\mathbf{S}^{(j)} = [S_0^{(j)}, S_1^{(j)}, \dots, S_{N-1}^{(j)}]^T = \text{DFT}(\mathbf{s}^{(j)})$ ($j = 1, 2, \dots, N_T$), we can describe the frequency-domain transmission rule as follows, for any subchannel k ($k = 0, 1, \dots, N-1$):

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{S}_k + \mathbf{N}_k, \quad (1)$$

where $\mathbf{S}_k = [S_k^{(1)}, S_k^{(2)}, \dots, S_k^{(N_T)}]^T$ is the "input vector", $\mathbf{N}_k = [N_k^{(1)}, N_k^{(2)}, \dots, N_k^{(N_R)}]^T$ is the Gaussian noise vector ($E[N_k^{(i)}] = 0$ and $E[|N_k^{(i)}|^2] = \sigma_N^2 = N_0 N$), \mathbf{H}_k denotes the $N_R \times N_T$ channel matrix with entries $H_k^{(i,j)}$, concerning a given channel realization, and $\mathbf{Y}_k = [Y_k^{(1)}, Y_k^{(2)}, \dots, Y_k^{(N_R)}]^T$ is the resulting, frequency-domain, "output vector".

As to a given MIMO channel realization, it should be noted that the Channel Frequency Response (CFR) $\mathbf{H}^{(i,j)} = [H_0^{(i,j)}, H_1^{(i,j)}, \dots, H_{N-1}^{(i,j)}]^T$, concerning the antenna pair (i, j) , is the DFT of the Channel Impulse Response (CIR) $\mathbf{h}^{(i,j)} = [h_0^{(i,j)}, h_1^{(i,j)}, \dots, h_{N-1}^{(i,j)}]^T$, where $h_n^{(i,j)} = 0$ for $n > L_s$ ($n = 0, 1, \dots, N-1$). Regarding a statistical channel model - which encompasses all possible channel realizations -, let us assume that $E[h_n^{(i,j)}] = 0$

and $E[h_n^{(i,j)*} h_{n'}^{(i,j)}] = 0$ for $n' \neq n$. By also assuming, for any (i, j, k) , a constant

$$E[|H_k^{(i,j)}|^2] = \sum_{n=0}^{N-1} E[|h_n^{(i,j)}|^2] = P_\Sigma \quad (2)$$

(of course, with $h_n^{(i,j)} = 0$ for $n > L_s$), the average bit energy at each BS antenna is given by

$$E_b = \frac{\sigma_s^2}{2\eta} P_\Sigma = \frac{\sigma_s^2}{2\eta N} P_\Sigma, \quad (3)$$

where $\eta = \frac{N}{N+L_s}$, $\sigma_s^2 = E[|S_k^{(j)}|^2]$ and $\sigma_s^2 = E[|s_n^{(j)}|^2] = \frac{\sigma_s^2}{N}$.

B. Linear Detection Techniques

An appropriate linear detector can be implemented by resorting to frequency-domain processing. After CP removal, a DFT operation leads to the required set $\{\mathbf{Y}_k; k = 0, 1, \dots, N-1\}$ of length- N_R inputs to the frequency-domain detector (\mathbf{Y}_k given by (1)); it works, for each k , as shown in Fig. 1(a), leading to a set $\{\tilde{\mathbf{Y}}_k; k = 0, 1, \dots, N-1\}$ of length- N_T outputs $\tilde{\mathbf{Y}}_k = [\tilde{Y}_k^{(1)}, \tilde{Y}_k^{(2)}, \dots, \tilde{Y}_k^{(N_T)}]^T$ ($k = 0, 1, \dots, N-1$).

When $N_T \leq N_R$, possibly with $N_R \gg 1$, either an MMSE, frequency-domain, optimum linear detection or a reduced-complexity, frequency-domain, linear detection can be considered. In all cases, the detection matrix, for each subchannel k ($k = 0, 1, \dots, N-1$) can be written as

$$\mathbf{D}_k = \mathbf{A}_k^{-1} \hat{\mathbf{H}}_k^H, \quad (4)$$

where $\hat{\mathbf{H}}_k^H$ is the conjugate transpose of the estimated MU-MIMO channel matrix $\hat{\mathbf{H}}_k$ and \mathbf{A}_k is a selected $N_T \times N_T$ matrix, possibly depending on $\hat{\mathbf{H}}_k$. Therefore, $\tilde{\mathbf{Y}}_k = \mathbf{D}_k \mathbf{Y}_k = \mathbf{A}_k^{-1} \hat{\mathbf{H}}_k^H \mathbf{Y}_k$ at the output of the frequency-domain linear detector (see Fig. 1(a)).

It should be noted that the j th component of $\hat{\mathbf{H}}_k^H \mathbf{Y}_k$ is given by $\sum_{i=1}^{N_R} \hat{H}_k^{(i,j)*} Y_k^{(i)}$ ($j = 1, 2, \dots, N_T$): this means that the $\hat{\mathbf{H}}_k^H$ factor provides N_T Maximal Ratio Combining (MRC) procedures, one per MT antenna, all of them based on an appropriate MF for each component of the length- N_R received vector at subchannel k .

For a MMSE detection - the optimum linear detection - or a Zero Forcing (ZF) detection [5], [11], an inversion of each $N_T \times N_T$ \mathbf{A}_k matrix ($k = 0, 1, \dots, N-1$) is required.

A reduced-complexity linear detection can be achieved by using an $N_T \times N_T$ diagonal matrix \mathbf{A}_k . The easiest implementation corresponds to adopting an identity matrix $\mathbf{A}_k = \mathbf{I}_{N_T}$. Of course, $\mathbf{D}_k = \hat{\mathbf{H}}_k^H$ and $\tilde{\mathbf{Y}}_k = \hat{\mathbf{H}}_k^H \mathbf{Y}_k$ when $\mathbf{A}_k = \mathbf{I}_{N_T}$, which means an "MF detection", actually not requiring a matrix inversion.

For a given channel realization \mathbf{H}_k and a given detection matrix \mathbf{D}_k , which depends on the estimated channel

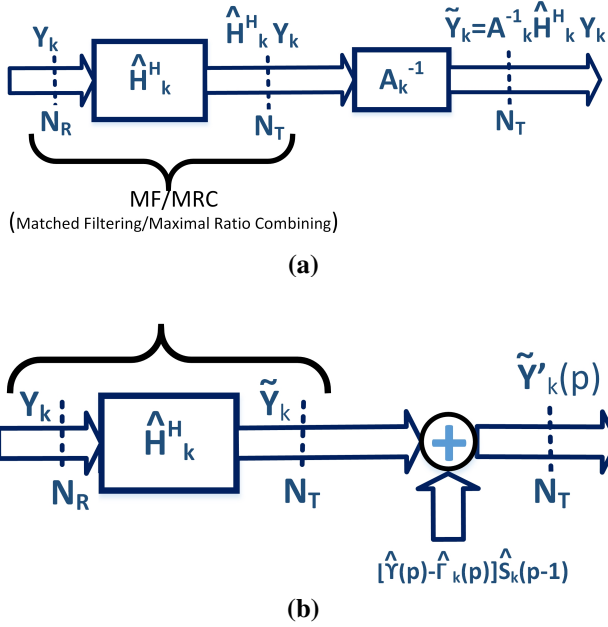


Fig. 1. Frequency-domain linear detection (a) and iterative DF detection combining a linear MF detection and interference cancellation in the frequency domain (b).

realization \hat{H}_k , the output of the frequency-domain detector is given by

$$\tilde{Y}_k = D_k Y_k = \Gamma_k S_k + N'_k, \quad (5)$$

where $\Gamma_k = D_k H_k$ and $N'_k = D_k N_k$.

With SC/FDE (time-domain data symbols), an Inverse Discrete Fourier Transform (IDFT) is required for each $\tilde{Y}^{(j)} = [\tilde{Y}_0^{(j)}, \tilde{Y}_1^{(j)}, \dots, \tilde{Y}_{N-1}^{(j)}]^T$ vector. The n th component of the resulting length- N IDFT $(\tilde{Y}^{(j)}) = \tilde{y}^{(j)}$ vector can be written as

$$\tilde{y}_n^{(j)} = \gamma^{(j)} s_n^{(j)} + ISI + MUI + 'Gaussian noise', \quad (6)$$

$$\text{with } \gamma^{(j)} = \frac{1}{N} \sum_{k=0}^{N-1} \Gamma_k^{(j,j)} \quad \left(\gamma^{(j)} = \frac{E[\tilde{y}_n^{(j)} s_n^{(j)*}]}{\sigma_s^2} \right).$$

Therefore, \tilde{Y}_k can be written as

$$\tilde{Y}_k = \gamma S_k + (\Gamma_k - \gamma) S_k + D_k N_k, \quad (7)$$

where γ is a diagonal $N_T \times N_T$ matrix with (j, j) entries given by $\gamma^{(j)} = \frac{1}{N} \sum_{k=0}^{N-1} \Gamma_k^{(j,j)}$.

When $\tilde{Y}_k^{(j)}$ is written as

$$\tilde{Y}_k^{(j)} = \gamma^{(j)} S_k^{(j)} + [\Gamma_k^{(j,j)} - \gamma^{(j)}] S_k^{(j)} + \sum_{l=1, l \neq j}^{N_T} \Gamma_k^{(j,l)} S_k^{(l)} + \sum_{i=1}^{N_R} D_k^{(j,i)} N_k^{(i)}, \quad (8)$$

the four terms in the right-hand side of eq. (8) are concerned, respectively, to "signal", ISI, MUI/MSI and "Gaussian noise", at subchannel k .

C. Low-Complexity Iterative DF Technique

A low-complexity iterative DF technique can be easily devised having in mind eq. (7). This frequency-domain nonlinear detection technique combines the use of a linear detector and, for all iterations after the initial iteration (i.e., for $p > 1$), a cancellation of residual MUI - and residual MSI, when some users adopt several TX antennas for spatial multiplexing purposes - as well as residual ISI; such cancellation is based on the estimated data block which is provided by the preceding iteration and fed back to the frequency-domain detector. The output of this frequency-domain detector, for iteration p , is as follows:

$$\tilde{Y}'_k(p) = \tilde{Y}_k + [\hat{\gamma}(p) - \hat{\Gamma}_k(p)] \hat{S}_k(p-1), \quad (9)$$

$[k = 0, 1, \dots, N-1; p > 1 \text{ (for } p = 1, \tilde{Y}'_k(p) = \tilde{Y}_k)]$, where $\hat{\Gamma}_k(p) = D_k(p) \hat{H}_k$ - with $D_k(p)$ denoting the detection matrix employed in iteration p - and the entries (j, j) of the diagonal matrix $\hat{\gamma}(p)$ are given by $\hat{\gamma}^{(j)}(p) = \frac{1}{N} \sum_{k=0}^{N-1} \hat{\Gamma}_k^{(j,j)}(p)$.

Of course, $[\hat{S}_0^{(j)}(p-1), \dots, \hat{S}_{N-1}^{(j)}(p-1)]^T = DFT \left([\hat{s}_0^{(j)}(p-1), \dots, \hat{s}_{N-1}^{(j)}(p-1)]^T \right)$.

The implementation of this iterative DF technique is especially simple when $D_k(p) = \hat{H}_k^H$ for any p , i.e., when a linear MF detector is adopted as shown in Fig. 1 (b) for all iterations; the matrix inversion which is inherent to more sophisticated linear detectors is then avoided. On the other hand, a slightly improved performance can be achieved through an increased implementation complexity, by feeding back vectors of soft (instead of hard) time-domain symbol decisions for interference cancellation.

III. EVALUATION OF THE ACHIEVABLE DETECTION PERFORMANCES

A. Semi-analytical Performance Evaluation

Regarding evaluation of detection performances by simulation, simple semi-analytical methods are presented here, for the detection techniques of subsecs II-B and II-C, both combining simulated channel realizations and analytical computations of BER performance which are conditional on those channel realizations. In all cases, the conditional BER values are directly computed by resorting to a Signal-to-Interference-plus-Noise-Ratio (SINR), under the assumption that the "interference" has a quasi-Gaussian nature. These ratios are simply derived in accordance with the channel realization H_k ($k = 0, 1, \dots, N-1$). Of course, for concluding the BER computation in each case - involving random generation of a large number of channel realizations and conditional BER computations - a complementary averaging operation over the set of channel realizations is required.

When using a linear detection technique (sec. II-B), it is easy to conclude, having in mind (8), that the "signal-to-interference-plus-noise" ratio concerning the j th input of the MU-MIMO system is given by

TABLE I
SIMO ($1 \times N_R$) PERFORMANCE BOUNDS.

SIMO/LDB	$SINR_1 = \frac{1}{\alpha} \frac{\sum_{k=0}^{N-1} \frac{\sum_{i=1}^{N_R} H_k^{(i,1)} ^2}{\alpha + \sum_{i=1}^{N_R} H_k^{(i,1)} ^2}}{\sum_{k=0}^{N-1} \frac{1}{\alpha + \sum_{i=1}^{N_R} H_k^{(i,1)} ^2}}$
SIMO/MFB	$SINR_1 = \frac{1}{\alpha N} \sum_{k=0}^{N-1} \sum_{i=1}^{N_R} H_k^{(i,1)} ^2$
SIMO/AWGN/MFB	$SINR_1 = 2\eta N_R \frac{E_b}{N_0}$

$$SINR_j = \frac{N|\gamma^{(j)}|^2}{\beta_j + \sum_{l=1}^{N_T} \beta_l + \alpha \sum_{i=1}^{N_R} \sum_{k=0}^{N-1} |D_k^{(j,i)}|^2} \quad (10)$$

where $\alpha = \frac{N_0}{\sigma_s^2}$, $\beta_j = \sum_{k=0}^{N-1} |\Gamma_k^{(j,j)} - \gamma^{(j)}|^2$ and $\beta_l = \sum_{k=0}^{N-1} |\Gamma_k^{(j,l)}|^2$ with $l \neq j$.

For 4-QAM transmission, the resulting BER_j ($j = 1, 2, \dots, N_T$) - conditional on the channel realization $\{\mathbf{H}_k; k = 0, 1, \dots, N-1\}$ - is given by

$$BER_j \approx Q\left(\sqrt{SINR_j}\right), \quad (11)$$

(where $Q(\cdot)$ is the Gaussian error function) with $SINR_j$ as computed above, and $BER = \frac{1}{N_T} \sum_{j=1}^{N_T} BER_j$.

When using the iterative DF technique of Fig. 1 (b), eq. (11) is replaced by $BER_j(p) \approx Q\left(\sqrt{SINR_j(p)}\right)$ for $p \geq 1$; certainly, $\mathbf{D}_k(p) = \hat{\mathbf{H}}_k^H$ and $\mathbf{\Gamma}_k(p) = \hat{\mathbf{H}}_k^H \mathbf{H}_k$ in the computation of $SINR_j(p)$. By assuming a perfect channel estimation ($\hat{\mathbf{H}}_k = \mathbf{H}_k$), this computation for $p > 1$ can be made especially simple, taking into account that $\tilde{\mathbf{Y}}'(p) = \gamma \mathbf{S}_k + (\mathbf{\Gamma}_k - \gamma) (\mathbf{S}_k - \hat{\mathbf{S}}_k(p-1)) + \mathbf{D}_k \mathbf{N}_k$ and $E\left(|s_n^{(j)} - \hat{s}_n^{(j)}(p-1)|^2\right) = 4\sigma_s^2 BER_j(p-1) \approx 4\sigma_s^2 Q\left(\sqrt{SINR_j(p-1)}\right)$. Therefore, we get

$$SINR_j(p) \approx \frac{N|\gamma^{(j)}|^2}{\beta_j + \sum_{l=1}^{N_T} \beta_l + \alpha \sum_{i=1}^{N_R} \sum_{k=0}^{N-1} |D_k^{(j,i)}|^2} \quad (12)$$

where $\beta_l = 4Q\left(\sqrt{SINR_l(p-1)}\right) \sum_{k=0}^{N-1} |\Gamma_k^{(j,l)}|^2$ with $l \neq j$ and $\beta_j = 4Q\left(\sqrt{SINR_j(p-1)}\right) \sum_{k=0}^{N-1} |\Gamma_k^{(j,j)} - \gamma^{(j)}|^2$.

B. Reference MMSE Performance and SIMO Performance Bounds

When adopting an "MMSE detector" and a perfect channel estimation is assumed, $\mathbf{D}_k = (\mathbf{H}_k^H \mathbf{H}_k + \alpha \mathbf{I}_{N_T})^{-1} \mathbf{H}_k^H$ [11]. It can be shown that the resulting $SINR_j$ - which can be used for computing BER_j , and then BER - can be written as $SINR_j = \frac{\gamma^{(j)}}{1-\gamma^{(j)}}$, with $\gamma^{(j)}$ as defined in subsec. II-B.

Successively improved performance bounds can be obtained as follows (see Table I): also under $\mathbf{D}_k = (\mathbf{H}_k^H \mathbf{H}_k + \alpha \mathbf{I}_{N_T})^{-1} \mathbf{H}_k^H$, with the same N_R but $N_T = 1$, which corresponds to a SIMO/Linear Detection Bound (LDB); under $\mathbf{D}_k = \mathbf{H}_k^H$, with $N_T = 1$ and the same N_R , by suppressing the resulting first term (ISI) in the denominator of eq. (10), which corresponds to a SIMO/MFB; when replacing the fading channel by an AWGN channel

for each $(i, 1)$ antenna pair ($i = 1, 2, \dots, N_R$) in the SIMO/MFB, which corresponds to a SIMO/AWGN/MFB.

It should be noted that the BER curve for the SIMO/AWGN/MFB actually corresponds to the achievable BER performance (against White Gaussian Noise) in a SIMO $1 \times N_R$ system with single-path propagation for all $(i, 1)$ antenna pairs, provided that an MF detection, under ideal channel estimation, is adopted:

$$BER = Q\left(\sqrt{2\eta N_R \frac{E_b}{N_0}}\right) \quad (13)$$

C. Massive MIMO effects

When $N_R \gg N_T$, both the MUI/MSI effects and the effects of multipath propagation (fading, ISI) tend to disappear: consequently, the BER performances for the MU-MIMO $N_T \times N_R$ Rayleigh fading channel become very close to those concerning a SIMO $1 \times N_R$ channel with single-path propagation for all N_R TX/RX antenna pairs. The achievable performances under a "truly massive" MU-MIMO implementation can be analytically derived as explained in the following.

Entries of \mathbf{H}_k are i.i.d. Gaussian-distributed random variables with zero mean and variance P_Σ . Therefore, $\lim_{N_R \rightarrow \infty} \left[\frac{1}{N_R} \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2 \right] = E\left[|H_k^{(i,j)}|^2\right] = P_\Sigma$ and

$$\begin{aligned} \lim_{N_R \rightarrow \infty} \left[\frac{1}{N_R} \sum_{i=1}^{N_R} \sum_{l \neq j}^{N_R} H_k^{(i,j)*} H_k^{(i,l)} \right] \\ = E_{l \neq j} \left[H_k^{(i,j)*} H_k^{(i,l)} \right] = 0. \end{aligned} \quad (14)$$

Consequently, for $N_R \gg N_T$,

$$\sum_{i=1}^{N_R} |H_k^{(i,j)}|^2 \approx N_R P_\Sigma \quad (15)$$

and

$$\sum_{i=1}^{N_R} \sum_{l \neq j}^{N_R} H_k^{(i,j)*} H_k^{(i,l)} \approx 0. \quad (16)$$

Therefore,

$$\lim_{N_R \rightarrow \infty} \left(\frac{SINR_j}{N_R} \right) = \frac{\sigma_s^2}{N_0} P_\Sigma = 2\eta \frac{E_b}{N_0} \quad (17)$$

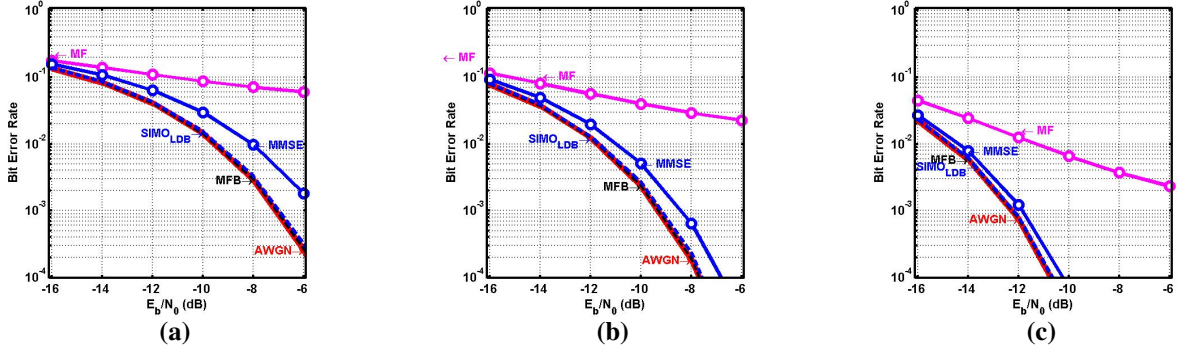


Fig. 2. BER performances for SC/FDE-based MU-MIMO, with $N_T = 10$ and $N_R = 30$ (a), 50 (b) or 100 (c), when using linear detection (MF and MMSE) [with the SIMO/LDB (dashed line), the SIMO/MFB and the SIMO/AWGN/MFB ($1 \times N_R$) performances also included].

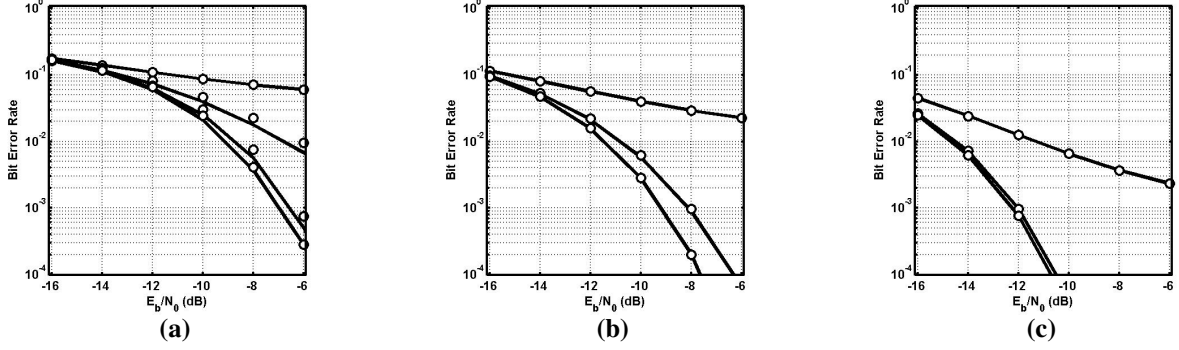


Fig. 3. BER performances, at the several iterations ($p = 1, 2, 3, 4$) for SC/FDE-based MU-MIMO, with $N_T = 10$ and $N_R = 30$ (a), 50 (b) or 100 (c), when using the iterative DF detection technique.

(by assuming that $\mathbf{D}_k = \mathbf{H}_k^H$). When $N_R \gg N_T$,

$$\text{SINR}_j \approx N_R \lim_{N_R \rightarrow \infty} \left(\frac{\text{SINR}_j}{N_R} \right) = 2\eta N_R \frac{E_b}{N_0}, \quad (18)$$

which implies that $\text{BER} \approx Q\left(\sqrt{2\eta N_R \frac{E_b}{N_0}}\right)$, i.e. a BER performance closely approximating the SIMO/AWGN/MFB (eq. (13)).

When $N_R \gg N_T$, it should also be noted - having in mind the equations (15) and (16) - that the MMSE linear detection becomes practically equivalent to a MF linear detection, since

$$\mathbf{D}_k = (\mathbf{H}_k^H \mathbf{H}_k + \alpha \mathbf{I}_{N_T})^{-1} \mathbf{H}_k^H \approx \beta \mathbf{H}_k^H, \quad (19)$$

with $\beta = \frac{1}{\alpha + N_R P_\Sigma}$ (assuming a perfect channel estimation). Of course, the corresponding $\mathbf{\Gamma}_k$ matrix is then $\mathbf{\Gamma}_k \approx \beta \mathbf{H}_k^H \mathbf{H}_k \approx \frac{N_R P_\Sigma}{\alpha + N_R P_\Sigma} \mathbf{I}_{N_T}$, leading to $\gamma^{(j)} \approx \Gamma_k^{(j,j)} \approx \frac{N_R P_\Sigma}{\alpha + N_R P_\Sigma}$ ($j = 1, 2, \dots, N_T$). Therefore, the resulting SINR's are as expected, under a perfect channel estimation ($\text{SINR}_j = \frac{\gamma^{(j)}}{1 - \gamma^{(j)}} \approx \frac{N_R P_\Sigma}{\alpha} = 2\eta N_R \frac{E_b}{N_0}$). Clearly, when $N_R \gg N_T$, the MUI/MSI effects as well as both the fading and the ISI effects of multipath propagation become vanishingly small, leading to a close approximation to the SIMO/AWGN/MFB reference performance.

IV. NUMERICAL RESULTS AND DISCUSSION

The set of performance results which are presented here are concerned to SC/FDE uplink block transmission, with

$N = 256$ and $L_s = 64$ in a MU-MIMO $N_T \times N_R$ Rayleigh fading channel. Perfect channel estimation and perfect power control are assumed. The fading effects regarding the several TX/RX antenna pairs are supposed to be uncorrelated, with independent zero-mean complex Gaussian $h_n^{(i,j)}$ coefficients assumed to have variances $P_n = 1 - \frac{n}{63}$, $n = 0, 1, \dots, 63$ ($P_n = 0$ for $n = 64, 65, \dots, 255$).

The accuracy of performance results obtained through the semi-analytical simulation methods of sec. III was assessed by means of parallel conventional Monte Carlo simulations (involving an error counting procedure), which correspond to the superposed dots in the several BER performance curves of Figs. 2 and 3, concerning the MU-MIMO system.

Fig. 2 shows the simulated BER performances for an SC/FDE-based MU-MIMO uplink and three possibilities regarding N_R for $N_T = 10$, when using two linear detection techniques: optimum (MMSE) detection; reduced-complexity (MF) detection. In each subfigure, for the sake of comparisons, we also include the SIMO $1 \times N_R$ performance bounds of Table I. In the simulation results concerning each subfigure of Fig. 2, the five BER performance curves are ordered, from the worst to the best, as follows: $N_T \times N_R$ MU-MIMO with reduced-complexity (MF) linear detection; $N_T \times N_R$ MU-MIMO with MMSE detection; SIMO/LDB ($1 \times N_R$); SIMO/MFB ($1 \times N_R$); SIMO/AWGN/MFB ($1 \times N_R$) [practically superposed to the SIMO/MFB curve]. These results clearly show that the

performance degradation which is inherent to the reduced-complexity linear detection technique (MF) - as compared with the MMSE linear detection - can be made quite small, by increasing N_R significantly; they also show that, under highly increased N_R values, the "MUI/MSI-free" SIMO (multipath) performance and the ultimate bound - the "MUI/MSI-free and ISI & fading-free" SIMO (single-path) performance - can be closely approximated, even when adopting the reduced-complexity linear detection. This figure emphasizes a "massive MIMO" effect when $N_R \gg N_T$, which leads to BER performances very close to the ultimate "MUI/MSI-free and ISI & fading-free" SIMO (single-path) performance bound (the SIMO/AWGN/MFB of Table I).

Fig. 3 shows the simulated BER performances for an SC/FDE-based MU-MIMO uplink and three possibilities regarding N_R for $N_T = 10$, when using the reduced-complexity iterative DF detection technique of Fig. 1 (b), which does not require matrix inversions. By comparing these results to the results of Fig. 2, also for $N_T = 10$, we can conclude that, whenever $N_R \geq 3N_T$, the reduced-complexity DF technique of Fig. 1 (b) is able to provide BER performances which are better than those of the MMSE linear detector [while avoiding the inversion of a 10×10 matrix A_k ($k = 0, 1, \dots, N - 1$)], closely approximating the (practically identical) SIMO/MFB and the SIMO/AWGN/MFB reference performances after a small number of iterations.

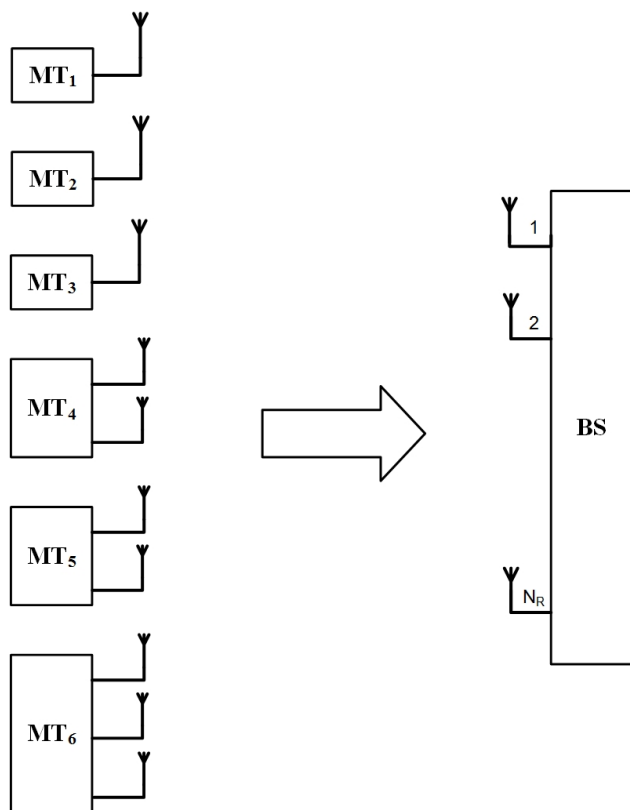


Fig. 4. Uplink transmission scenario with six users and $N_T = 10$ TX antennas.

It should be noted that the numerical results reported above, for $N_T = 10$ TX antennas, are compatible with an uplink transmission scenario involving up to 10 users (for example, 10 users with one TX antenna per user); a specific scenario with $N_T = 10$ TX antennas, involving six users, is depicted in Fig. 4.

V. CONCLUSIONS

This paper was dedicated to the uplink detection and performance evaluation for a MU-MIMO system with SC/FDE transmission, when adopting a large number of antennas and low-complexity detection techniques at the BS. With the help of selected numerical performance results, discussed in detail in Section IV, we show that a moderately large number of BS antennas (say, $N_R = 3N_T$) is enough to closely approximate the SIMO/MFB performance - and also the SIMO/AWGN/MFB performance, expressed as $BER = Q\left(\sqrt{2\eta N_R \frac{E_b}{N_0}}\right)$, especially when using the suggested low-complexity iterative DF technique, which does not require $N_T \times N_T$ matrix inversion. We also emphasize the "massive MIMO" effects provided by a number of BS antennas much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users, even when strongly reduced-complexity linear detection techniques - such as the so-called "MF detection -, are adopted.

The accuracy of performance results obtained by semi-analytical means, much less time-consuming than conventional, 'error counting'-based, Monte Carlo simulations - was also demonstrated. The proposed performance evaluation method can be very useful for rapidly knowing "how many antennas do we need in the BS?", for a given number of antennas jointly employed in the user terminals.

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